Journal of Basic and Applied Engineering Research
p-ISSN: 2350-0077; e-ISSN: 2350-0255; Volume 3, Issue 13; October-December, 2016, pp. 1152-1158

http://www.krishisanskriti.org/Publication.html

Finite Time Continuous Terminal Sliding Mode Control for Trajectory Tracking of Robotic Manipulator

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Abstract—*This paper investigates a Finite Time Continuous Terminal Sliding Mode control for Trajectory Tracking of robotic manipulators. Robustness and convergence results are presented for trajectory tracking control of 2dof and 3dof robotic manipulators in presence of nonliniarities, bounded uncertainties and unknown external disturbances. The effectiveness of the controller is evaluated in Matlab environment. From the results it is observed that fast convergence, highly precise tracking can be obtained by using the controller which is crucial for the robot handling and stability. It is established by applying the controller for trajectory tracking control of industrial robotic manipulator PUMA 560. The chattering problem is also fairly handled giving a smooth tracking trajectory.*

Index Terms*: Finite time convergence; Robustness; erminal sliding mode control; PUMA 560 Robotic manipulator.*

1. INTRODUCTION

In spite of the innumerable existing commercial robots, robotic manipulator motion control is still a field of rigorous study and research. The Robotic manipulators are usually used in the environments where human access is difficult, dangerous to health, and to handle materials without direct human contact. Typical industrial manipulator applications like welding, painting, pick and place, packaging, product inspection etc are accomplished with high precision and speed. Robotic manipulators are multiple input multiple out- put (MIMO) systems[9,10] with highly nonlinear dynamics. This makes the trajectory tracking control of robotic manip- ulators a challenging task. The system identication of robot is very difficult. As a result there are always uncertainties due to system dynamics which degrade the performance of the controller. In many applications robot manipulators are used in an unknown and unstructured environment. There- fore uncertainties due to modeling error, parametric variation and external disturbances are always present in practical robotic applications. Therefore design of nonlinear robust controllers[9,10] with strong mathematical tools is a prime requirement for trajectory tracking of robotic manipulator. The controller should guarantee asymptotic error convergence andstability in presence of such uncertainties Thus the trajectory control of robotic manipulator requires control schemes that take nonlinearities of the system, modeling uncertainties as well as external disturbances into account[10].

Sliding mode control(SMC)[2,8,9] is one of the most appro- priate approach for control of robotic manipulators. It has attracted signifcant amount of interest due to its fast global convergence, simplicity of implementation, order reduction, high robustness to external disturbances and insensitivity to model errors and system parameter variations.Control of robotic manipulators using sliding mode control has a rather long history. Numerous variations have been proposed in the literature[1,2,3,5,9].

In conventional sliding mode switching manifolds are usually linear hyper planes which guarantee asymptotic stability[2,8,9]. However for faster error convergence, the sliding mode controller parameters should be chosen such that the poles of the sliding mode dynamics are far from the origin on the left half of the s-plane. But this will cause increase of gain of the controller which may cause severe chattering on the sliding motion and thus deteriorates the system performance. To solve this problem of global asymptotic stability, terminal sliding mode control (TSMC) scheme has been developed [3,4,5] to achieve finite time stabilization. The TSM Controller was originated from the concept, terminal attractors[3]. The TSMC was first used in[6] for finite time sliding mode control design for robotic manipulators. It was then extended to different control problems of SISO and MIMO systems including robotics[3-7,11,12]. This paper examines the application of TSMC on trajectory tracking control of robotic manipulators. The outline of this paper is as follows. Section 2 explains the Finite Time Continuous Terminal Sliding Mode Controller. Section3 explains the Tracking Control of Robotic Manipula- tor using TSMC. Section4 explains the Simulation Example. Simulation is done for both 2dof as well as 3dof PUMA 560 manipulator. Conclusions are drawn in Section5.And APPENDIX A describes the dynamics of 2 dof manipulator. APPENDIX B presents the simulation results of 2 dof robotic manipulator and APPENDIX C presents the simulation results of PUMA 560 robotic manipulator.

2. THE FINITE TIME CONTINUOUS TERMINAL SLIDING MODE CONTROLLER(TSMC)

The TSMC as proposed by S.Yu, X.Yu, B.S. and Z. man [2005] [6] is discussed below. The following first-order nonlinear differential equations describe the TSM and Fast TSM as:

$$\sigma = \dot{z} + \beta |z|^{\gamma} sign(z) = 0$$
$$\sigma = \dot{z} + \alpha z + \beta |z|^{\gamma} sign(z) = 0 \quad (1)$$

respectively, where $z \in R, \alpha, \beta > 0, 0 < \gamma < 1$.

The equilibrium point $z = 0$ of the above equation (1) is globally finite-time stable, i.e., for any given initial condition $z(0) = z_o$, the system state converges to $z = 0$ in finite time T as given below for TSM and FTSM

$$T = \frac{1}{\beta(1-\gamma)} |z_o|^{1-\gamma}$$
$$T = \frac{1}{\alpha(1-\gamma)} \ln \frac{\alpha |z_o|^{1-\gamma} + \beta}{\beta} \quad (2)$$

respectively and stays there forever, i.e. $z = 0$ for $t > T$

An extended Lyapunov description of finite time stability can be given with the form of fast TSM as

$$\dot{V}(z) + \alpha V(z) + \beta V^{\gamma}(z) \le 0, 0 < \gamma < 1 \quad (3)$$

and the settling time can be given by

$$T \le \frac{1}{\alpha(1-\gamma)} \ln \frac{\alpha V^{1-\gamma}(z_o) + \beta}{\beta} \quad (4)$$

Thus the above equations (3) and (4) mean exponential stability and faster finite-time stability. And the NTSM can be expressed as

$$\sigma = z + \beta |\dot{z}|^{\gamma} sign(\dot{z}) = 0 \quad (5)$$

Where $\beta > 0$ and $1 < \gamma < 2$. Although the absolute value and signum operators are involved it is continuous and differentiable. Its first derivative can be expressed as

$$\dot{\sigma} = \dot{z} + \beta\gamma |\dot{z}|^{\gamma-1} \ddot{z} \quad (6)$$

3. TRACKING CONTROL OF ROBOTIC MANIPULATOR USING TERMINAL SLIDING MODE CONTROL (TSMC)

Here The Fast Continuous Terminal Sliding Mode controller [6] is developed for trajectory tracking of robotic manipulators with the TSMC as discussed in section II. Precise trajectory tracking can be achieved with faster finite time convergence as compared to the conventional continuous sliding mode control.

3.1 Controller design

As we know the n link rigid robotic manipulator dynamics [9,10] can be given as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \quad (7)$$

Where $q \in R^n$ denotes the joint angle vector, $\dot{q}$ and $\ddot{q}$ are the joint angular velocity and the joint angular acceleration respectively. And $M(q) \in R^{n\times n}$ denotes the inertia matrix and $C(q,\dot{q}) \in R^{n\times n}$ denotes the centrifugal and Coriolis matrix. Also, $G(q) \in R^n$ denotes the gravitational torque vector acting on the joints. The joint torque vector $\tau \in R^n$ is the control input to the system.

And let $q_d \in R^n$ be a twice differentiable desired trajectory, also define the tracking error as $e = q - q_d$. The control objective is to determine a feedback control torque τ such that the manipulator output q tracks the desired trajectory q_d faithfully in finite time.

The following notions are used for simplicity of expression in developing TSM (Haimo 1986)[17]:

$$y^{\gamma} = [y_1^{\gamma_1}, ..., y_n^{\gamma_n}]^T$$
$$|y|^{\gamma} = [|y_1|^{\gamma_1}, ..., |y_n|^{\gamma_n}]^T$$
$$sig(y)^{\gamma} = [|y_1|^{\gamma_1} sign(y_1), ..., |y_n|^{\gamma_n} sign(y_n)]^T$$

Hence, the TSM can be defined as

$$\sigma = e + \beta sig(\dot{e})^{\gamma} = 0 \quad (8)$$

where $\sigma = [\sigma_1, ..., \sigma_n]^T \in R^n$, $\beta = diag(\beta_1, ..., \beta_n)$, and $1 < \gamma_1, ..., \gamma_n < 2$

$$\dot{\sigma} = \dot{e} + \beta diag(\gamma_1 |\dot{e}_1|^{\gamma_1 - 1}, ..., \gamma_n |\dot{e}_n|^{\gamma_n - 1})$$
$$(M(q)^{-1}(\tau - C(q,\dot{q})\dot{q} - G(q)) - \ddot{q}_d) \quad (9)$$

The conventional TSM control can be designed as a discontinuous control law according to a discontinuous reaching law such as

$$\dot{\sigma} = -ksign(\sigma) \tag{10}$$

where $k = diag(k_1, ..., k_n)$, $i = 1, ..., n$ and $sign(\sigma) = [sign(\sigma_1), ..., sign(\sigma_n)]^T$.

A discontinuous TSM control can be designed as

$$\begin{aligned} \tau &= C(q,\dot{q})\dot{q} + G(q) - M(q) \\ &(ksign(\sigma) - \ddot{q}_d + \beta^{-1}\gamma^{-1}\left|\dot{e}\right|^{2-\gamma}) \end{aligned} \tag{11}$$

Retaining the property of finite time reaching of TSM but eliminating discontinuities, [6] has proposed a Continuous Fast TSM type reaching law as

$$\dot{\sigma} = -k_1\sigma - k_2 sig(\sigma)^{\rho} \tag{12}$$

Where $k_1 = diag(k_{11}, ..., k_{1n})$, $k_2 = diag(k_{21}, ..., k_{2n})$, $k_{1i}, k_{2i} > 0, 0 < \rho = \rho_1 = ... = \rho_n < 1$. The inverse dynamics controller is designed as

$$\begin{aligned} \tau &= C(q,\dot{q})\dot{q} + G(q) - M(q) \\ &(k_1\sigma + k_2 sig(\sigma)^{\rho} - \ddot{q}_d + \beta^{-1}\gamma^{-1}\left|\dot{e}\right|^{2-\gamma}) \end{aligned} \tag{13}$$

This control law is continuous therefore is chattering free. And it does not involve any negative fractional power, hence it is also singularity [5,6] free.

3.2 Stability analysis

The Lyapunov function is considered as $V = \frac{1}{2}\sigma^T\sigma$. By differentiating V with respect to time,

$$\begin{aligned} \dot{V} &= \sigma^T\dot{\sigma} \\ &= \sigma^T[\dot{e} + \beta\gamma diag(\left|\dot{e}\right|^{\gamma-1})\ddot{e}] \\ &= \sigma^T[\dot{e} + \beta\gamma diag(\left|\dot{e}\right|^{\gamma-1}) \\ &\quad (M(q)^{-1}(\tau - C(q,\dot{q})\dot{q} - G(q)) - \ddot{q}_d)] \\ &= \sigma^T[-\beta\gamma diag(\left|\dot{e}\right|^{\gamma-1})k_1\sigma - \beta\gamma diag(\left|\dot{e}\right|^{\gamma-1})k_2 sig(\sigma)^{\rho}] \\ &= -\sigma^T K_x\sigma - \sigma^T K_y sig(\sigma)^{\rho} \\ &< 0 \end{aligned} \tag{14}$$

Where $K_x = \beta\gamma diag(\left|\dot{e}\right|^{\gamma-1})k_1 \in R^{n\times n}$ and $K_y = \beta\gamma diag(\left|\dot{e}\right|^{\gamma-1})k_2 \in R^{n\times n}$ are positive definite diagonal matrices.

The $\dot{V} < 0$ implies the stability in Lypunov sence. Now if $\dot{e} \neq 0$ from equation (5.14) we have

$$\dot{V} \leq -2k_x V - 2^{(\rho+1)/2}k_y V^{(\rho+1)/2}, \tag{15}$$

Where k_x and k_y represent the minimum eigenvalues of K_x and K_y respectively, and $1/2 < (\rho+1)/2 < 1$.

So according to the finite-time stability criterion (3), TSM (6) will be reached in the finite time i. e. the actual states track the desired trajectory in finite time.

4. SIMULATION EXAMPLE

4.1 For 2DOF robotic manipulator :

In this paper a planar 2dof RR manipulator as taken from [6,7], is used to demonstrate the given control approach. The manipulator and the associated variables are shown in Figure 1. And the dynamics of the two-link manipulator is given in appendix A. Let the reference trajectories are[6,7,11]:

$$\begin{aligned} q_{d_1} &= 1.25 - (7/5)e^{-t} + (7/20)e^{-4t} \\ q_{d_2} &= 1.25 + e^{-t} - (1/4)e^{-4t} \end{aligned} \tag{16}$$

and the initial states are selected as $q_1(0) = 0.4, q_2(0) = 1.8, \dot{q}_1 = 0$ and $\dot{q}_2 = 0$.

For experimental study it is assumed that an unknown load is carried by the robot as a part of the second link, then the parameters m_2, l_2 and I_2 will change to $m_2 + \Delta m_2, l_2 + \Delta l_2$ and $I_2 + \Delta I_2$ respectively. We also assume that the change Δm_2 is as *sine* of q_2. Also let the variation of parameters change in the intervals.

$$\begin{aligned} 0 &\leq \Delta m_2 \leq 0.5\sin(q_2) \\ 0 &\leq \Delta l_2 \leq 0.25 \\ 0 &\leq \Delta I_2 \leq 0.5 \end{aligned} \tag{17}$$

A Band Limited White Noise with noise power [0.1] is added to the control signal as an external disturbance. In order to eliminate the chattering problem ,the boundary layer method is used i.e. The $\text{sgn}(s)$ is replaced by $sat(s/\Delta)$ function, where Δ is boundary layer.

4.1.1 Controller design parameters *:* The given continuous TSM controller design parameters as from (6), (10), (11) are chosen as below to achieve best controller performance :

$\beta = diag(15,15), \gamma = 11/9, k_1 = k_2 = diag(5,5)$ and $\rho = 2/5$.

The simulation results are given in APPENDIX B.

4.2 For PUMA 560 robotic manipulator :

Also the above mentioned controller is used for trajectory tracking of PUMA-560 robot manipulator.

For PUMA-560 robot dynamics and all physical parameters one can reffer [14,16].

PUMA 560 robot model is obtained from MATLAB software Robotic Toolbox [13]. For simulation purpose only 3 joints of PUMA 560 i.e. joint1, joint2 and joint3 are considered. The joint axes of joints 4, 5 and 6 intersect at a common point and $q_4 = q_5 = q_6 = 0$.

The simulation is carried out for the trajectory demand as given below:

$$\begin{aligned} q_{d_1} &= 1.25 - (7/5)e^{-t} + (7/20)e^{-4t} \\ q_{d_2} &= 1.25 + e^{-t} - (1/4)e^{-4t} \\ q_{d_3} &= 1.25 + e^{-t} - (1/4)e^{-4t} \end{aligned} \tag{18}$$

And $q_1(0) = 0.4, q_2(0) = 1.8, q_3(0) = 1.8$ and $\dot{q}_1 = 0$, $\dot{q}_2 = 0, \dot{q}_3 = 0$.

To discuss the performance of the controller under uncertainties let the following changes are done in inertia matrix[14,16].

$$\begin{aligned} a_{11} &= a_{11} + 1.5; \\ a_{22} &= a_{22} + 3; \\ a_{33} &= a_{33} + 0.5; \end{aligned} \tag{19}$$

A Band Limited White Noise with noise power [0.1] is added to the control signal as an external disturbance.

4.2.1 Controller design parameters : The TSM controller design parameters as from (6),(10),(11) are chosen heuristically to achieve best controller performance as:

$\beta = diag(21, 9, 20), \gamma = 11/9, k_1 = k_2 = diag(10, 20, 10)$ and $\rho = 2/5$.

Simulation Results are given in APPENDIX C.

From simulation results it is clear that the controller track the desired trajectory faithfully with very low rise time. Same is true for other results. Faster convergence of errors and sliding surfaces are clear from their respective plots. From control input plots it is seen that no chattering is evident in the figures. To discuss the controller performance, the output performance parameter integrated absolute error (IAE) and the input performance parameters total variation (TV) and control energy by using 2-norm are calculated from the results. Also the time of convergence of errors and sliding surfaces for different joints are calculated. The results are given in Table II and Table III. These results prove the superiority of the TSM controller in all aspects.

Table I: TSMC Performance (for 2DOF Manipulator)

Controller Performance					
Joint	**IAE**	**TV**	**2-Norm**	**Convergence of s (sec)**	**Convergence of e (sec)**
Joint 1	3.1254	28.2	107.88	0.2	0.52
Joint 2	3.5105	106.5	115.9	0.2	0.52

Table II: TSMC Performance (for PUMA 560 Manipulator)

Controller Performance					
Joint	**IAE**	**TV**	**2-Norm**	**Convergence of s (sec)**	**Convergence of e (sec)**
Joint 1	3.653	97.7	96.5	0.6	0.8
Joint 2	5.942	586.6	437.9	0.6	0.8
Joint 3	4.415	183.2	190.5	0.6	0.8

5. CONCLUSION

A Finite Time Continuous Terminal Sliding Mode Controller for trajectory tracking of robotic manipulator is investigated in this paper where the terminal sliding manifold guarantees fast and finite time convergence. The TSM controller is successfully applied for 2 dof plannar RR manipulator and also for PUMA 560 manipulator systems which are affected by matched bounded uncertainities and external disturbances and guarantees finite time convergence of error ensuring satisfactory stabilization as well as tracking performances. All simulation results prove the effectiveness of the TSMC for robot handling and stability. The given control law is chattering free as well as singularity free.

APPENDIX A

The dynamic model of the two-link manipulator [9,10] can be given as follows

$$M(q) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{20}$$

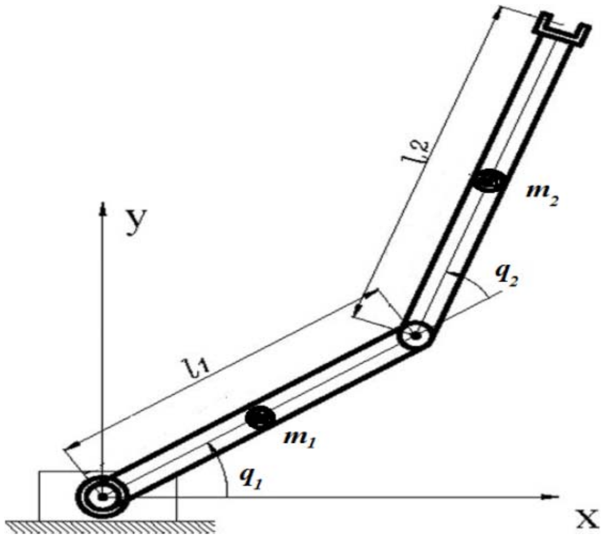


Fig. 1: Two link robotic manipulator with link masses m_1 and m_2 and link lengths l_1 and l_2 .

$$a_{11} = (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos(q_2) + J_1$$
$$a_{12} = m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2)$$
$$a_{21} = m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2)$$
$$a_{22} = m_2 l_2^2 + J_2$$

$$C(q,\dot{q})\dot{q} = \begin{bmatrix} -m_2 l_1 l_2 \sin(q_2)\dot{q}_2^2 - 2m_2 l_1 l_2 \sin(q_2)\dot{q}_1\dot{q}_2 \\ m_2 l_1 l_2 \sin(q_2)\dot{q}_2^2 \end{bmatrix}$$

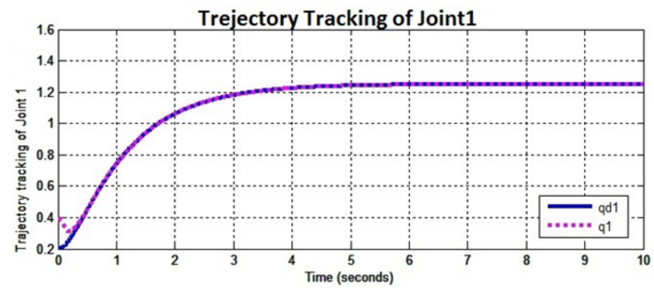
$$G(q) = \begin{bmatrix} (m_1 + m_2)l_1 g\cos(q_1) + m_2 l_2 g\cos(q_1 + q_2) \\ m_2 l_2 g\cos(q_1 + q_2) \end{bmatrix} \quad (21)$$

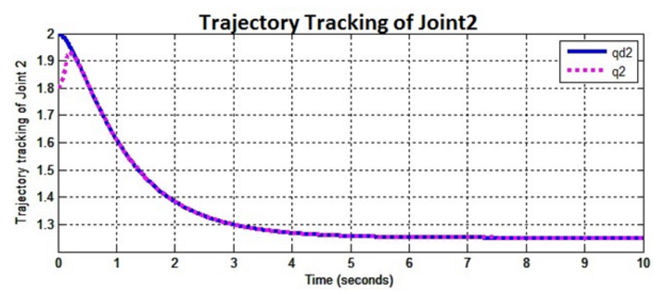
Here $q(t) = [q_1(t), q_2(t)]^T$ is the angular position vector where $q_1(t)$ and $q_2(t)$ are the angular positions of joints 1 and 2. $M(q)$ is the inertia matrix, $C(q,\dot{q})$ is the centripetal Coriolis matrix, $G(q)$ is the gravity vector and $\tau = [\tau_1, \tau_2]^T$ is the applied torque. Friction terms are ignored. Table I lists the physical parameters of the manipulator considered in the simulation study [6,7].

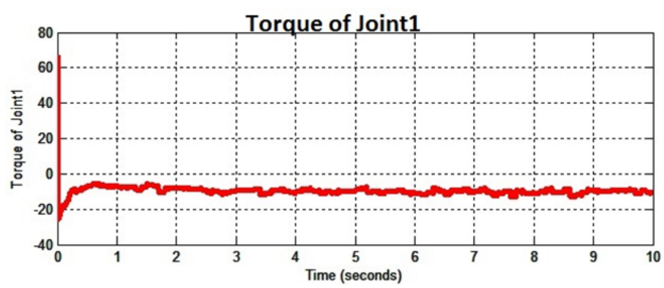
Table III: Physical Parameters of the 2-Link Robotic Manipulator[7]

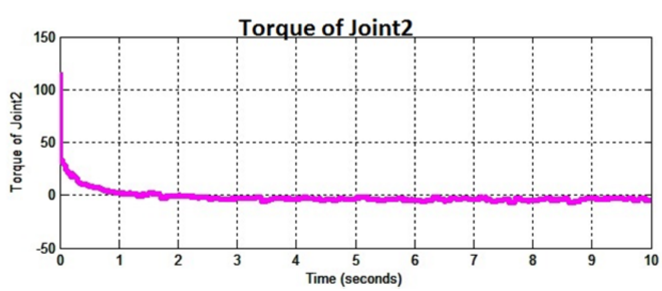
Symbol	Definition	Value
l_1	Length of the first link	1m
l_2	Length of the second link	0.85m
J_1	Moment of inertia of the D.C. motor 1	5 Kg.m^2
J_2	Moment of inertia of the D.C. motor 2	5 Kg.m^2
m_1	Mass of the link 1	0.5 kg
m_2	Mass of the link 2	1.5 kg
$\hat{m}_1$	Nominal Mass of the link 1	0.4 kg
$\hat{m}_1$	Nominal Mass of the link 2	1.2 kg
g	Gravitational constant	9.81 m/s^2

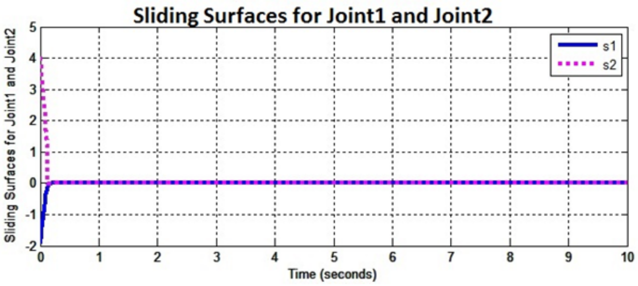
APPENDIX B : RESULTS OF 2 DOF MANIPULATOR

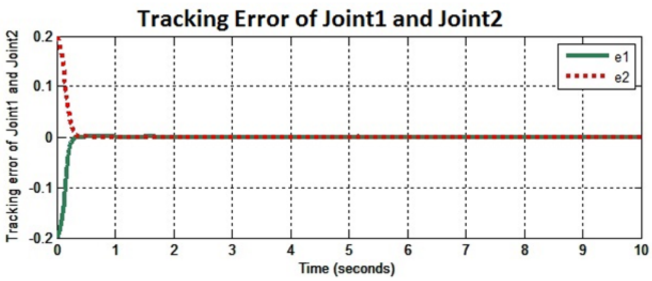


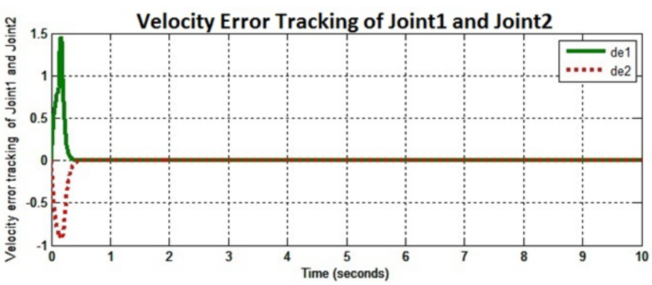




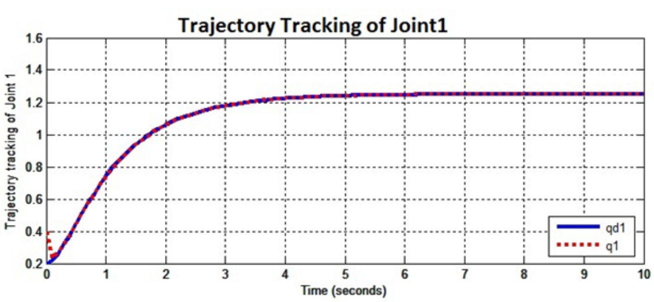


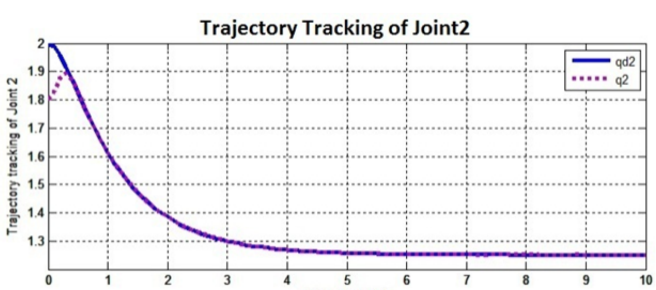


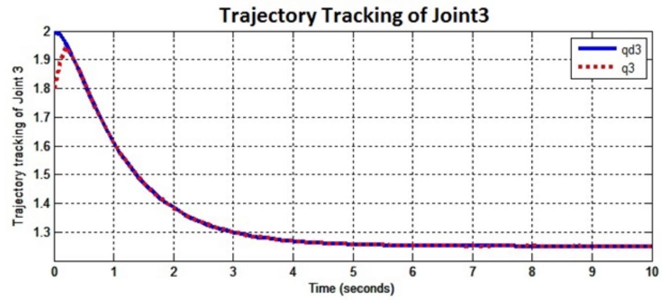


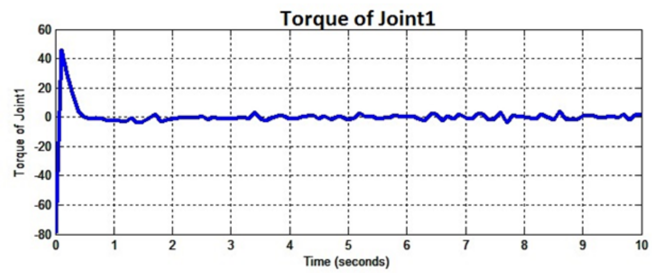


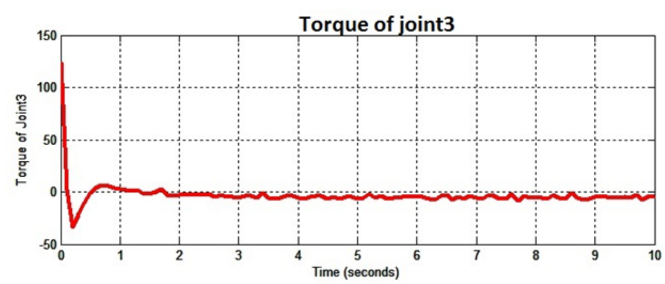
APPENDIX C: RESULTS OF PUMA 560

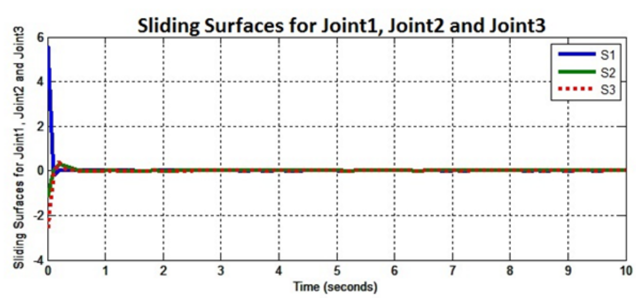


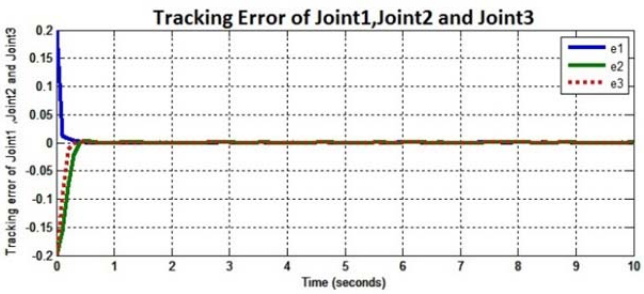


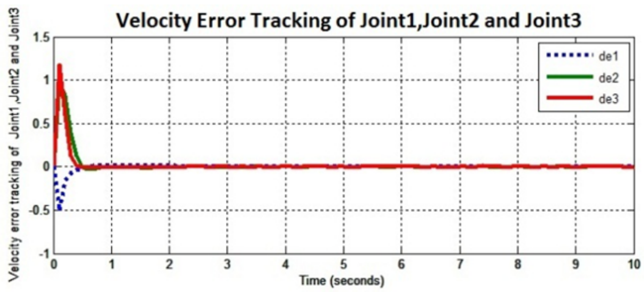












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